Equations in Physics

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Curriculum Connection: Physics, all grade levels.

Everywhere that physics is discussed it is safe to presume that the language of mathematics is never too far away. If we were to eavesdrop on these conversations, whether they occur in front of a blackboard or seated at a cafe with large enough napkins, we would not be surprised to hear and see the most instantly recognizable part of the language of mathematics: equations.

At the same time, it is well known that equations strike fear into the hearts of millions of our fellow citizens. I by no means exempt myself from this observation. I have grown accustomed to many of the more familiar equations, but perusing a text in an area of physics or mathematics that is new to me can still provoke a quiet feeling of uneasiness, even anxiety. I sometimes catch myself switching from reading a text closely, to scanning it rapidly, to closing the file in fairly quick succession, like a rock climber at first losing his grip, then sliding more and more uncontrollably down the page. Again, in the acknowledgments of his book *A Brief History of Time*, Stephen Hawking writes that "Someone told me that each equation I included in the book would halve the sales." [Hawking 1988] He still included one.

I want to focus here on just a few observations about equations that I plan to share with my students. When we learn to speak, we go through a fairly long process of trying out words, phrases and sentences in interaction and communication with our parents, our family, and our peers. In this process we make a number of mistakes. Particularly troublesome are those words that have multiple meanings depending on tone and context. I'm not only speaking of those words that do double or triple duty, and which have multiple definitions if you look them up in the dictionary. I'm referring also to extremely simple sentences like "Hey." It is of enormous importance in functioning well with others, socially, to be able to understand that sometimes this is a greeting, on the same level as "hi" or "how's it going?" and sometimes it is a command to get someone's attention, perhaps as a prelude to some longer phrase of moral instruction: "Hey! You shouldn't be standing there."

Similarly in physics, to someone new to the subject, it can be difficult to realize that many equations, seemingly identical in appearance or behavior, actually perform very different roles, and thus have very different meanings. Let's take a closer look at some of the equations that come up when the topic of forces and Newton's Laws is introduced.

$F = ma$

It is probably impossible to write a few words about this equation that would not be controversial to all readers.

Nevertheless, it seems fair to say that the "F" on the left hand side isn't a particular force – it stands for the net force, no matter what the physical reason for each of the forces that are being added up to get the net force. Also, this is really a vector equation, so students have to be reminded that this equation stands for one equation in the x direction, one in the y, and so on.

$F_{net} = 0$

The zero here doesn't imply that there are no forces at all, only that the net force is zero. There is a big difference between a situation where a person has no expenses and no income, and a situation where a person has large expenses and an income that exactly matches those expenses!

$F_{air} = 0$

The zero here doesn't really imply that the force due to air resistance is precisely zero, but rather that this force is so small in comparison to other numbers in the problem that we approximate it as equal to zero. A great deal of the practice of physics is exactly the thinking required to be able to say that some quantity can be ignored because you have argued that its effect is so small that it is negligible.

$F_N = mg$

This is a good example of an equation which is actually only true when the following two assumptions are made: that the normal force in question is acting on a mass that is resting on a horizontal surface, and that no other forces with a vertical component (other than gravity) are acting on the object. Such "contingent" equations occur quite often in physics, and it can be a sobering experience to realize that an equation that one has been using as if it were a stand-alone truth, actually has built into it some assumptions that no longer hold in the situation at hand.

One could profitably continue this list. There are equations that actually serve as definitions of the expression appearing on the left hand side. There are equations that are actually identities. (It is of course not an accident that notations have been developed replacing the ubiquitous "equal sign" when we are actually defining a variable, or writing down an identity). There are equations that are only true at one moment in an entire motion, while others hold true throughout the motion.

There is much more to physics than a mastery of equation-manipulation, but it is difficult to master physics without an attentiveness to these sorts of distinctions in equations. Some might argue that to call explicit attention to such distinctions will serve only to confuse students; there is already so much to learn, why add more to the plate? That might well be true, and it would be easy to go overboard with this approach. Still, most physics teachers have probably had the experience of marking a test where a student has "downloaded" onto the page those equations that he or she hopes to be most relevant, and yet there is no thread connecting one equation to the next. Perhaps a more self-conscious approach to the meanings of equations might help this situation.

References

Stephen W. Hawking. *A Brief History of Time*. N.Y.: Bantam 1988.